ALL SAINTS' COLLEGE

WA Exams Practice Paper C, 2016

Question/Answer Booklet

MATHEMATICS METHODS UNITS 3 AND 4

Section One: Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(4 marks)**

The gradient function of a curve is given by $f'(x) = \dfrac{48(6x+5)^3}{3}$.

Determine $f(x)$, given that the curve passes through the point (-1, 1).

$$y = (6x+5)^4 \Rightarrow y' = 24(6x+5)^3$$

$$f'(x) = \frac{2}{3} \times 24(6x+5)^3$$

$$f(x) = \frac{2}{3}(6x+5)^4 + c$$

$$f(-1) = \frac{2}{3}(-1)^4 + c$$

$$1 = \frac{2}{3} + c$$

$$c = \frac{1}{3}$$

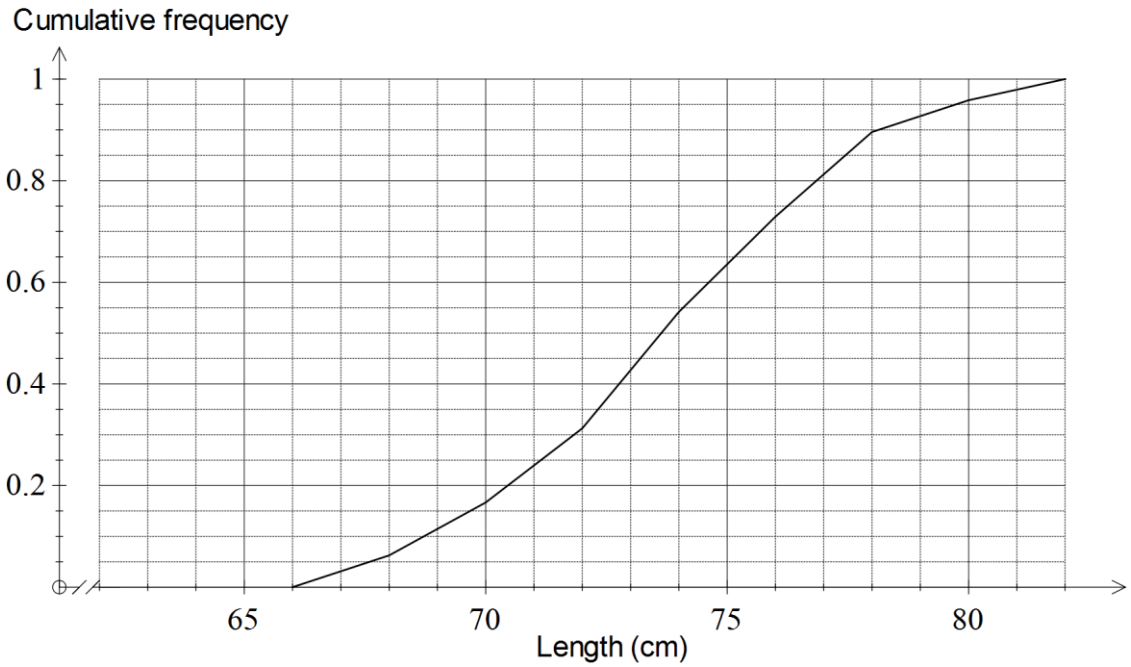
$$f(x) = \frac{2}{3}(6x+5)^4 + \frac{1}{3}$$

Question 2 **(7 marks)**

The lengths of a sample of 50 salmon caught by researchers from a fish farm are summarised in the following table.

Length (x cm)	Frequency	Relative frequency	Cumulative frequency
$66 < x \leq 68$	3	0.06	0.06
$68 < x \leq 70$	6	0.12	0.18
$70 < x \leq 72$	6	0.12	0.30
$72 < x \leq 74$	11	0.22	0.52
$74 < x \leq 76$	9	0.18	0.70
$76 < x \leq 78$	8	0.16	0.86
$78 < x \leq 80$	4	0.08	0.94
$80 < x \leq 82$	3	0.06	1.00

(a) Complete the relative and cumulative frequency columns above and then display the data as a cumulative frequency graph on the axes below. (4 marks)



(b) Use the graph and table above to estimate the following probabilities, where X is the length of a randomly caught salmon from the same fish farm.

(i) $P(70 < X < 76)$. (1 mark)

$0.12 + 0.22 + 0.18 = 0.52$ or $0.70 - 0.18 = 0.52$

(ii) $P(72.5 < X < 77.5)$. (2 marks)

Interpolating from graph: $0.85 - 0.37 = 0.43$

Question 3 **(8 marks)**

(a) Calculate the area under the curve $y = 3\cos\left(\frac{x}{4}\right)$ between $x = 0$ and $x = \frac{2\pi}{3}$. (3 marks)

$$A = \int_0^{\frac{2\pi}{3}} 3\cos\left(\frac{x}{4}\right) dx$$

$$= \left[12\sin\left(\frac{x}{4}\right)\right]_0^{\frac{2\pi}{3}}$$

$$= 12\sin\left(\frac{\pi}{6}\right) - 12\sin(0) = 6 \text{ sq units}$$

(b) Determine the area of the region enclosed between the line $y = 6x + 9$ and the curve $y = 3x^2$. (5 marks)

$$3x^2 = 6x + 9$$
$$3x^2 - 6x - 9 = 0$$
$$3(x^2 - 2x - 3) = 0$$

$$3(x-3)(x+1) = 0 \Rightarrow x = -1,\ x = 3$$

$$A = \int_{-1}^{3} 6x + 9 - 3x^2 \ dx$$

$$= \left[3x^2 + 9x - x^3\right]_{-1}^{3}$$

$$= (27 + 27 - 27) - (3 - 9 + 1)$$
$$= 32 \text{ sq units}$$

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Question 4 **(5 marks)**

Determine

(a) the equation of the asymptote of the graph of $y = \log_e(x - e)$. (1 mark)

$x = e$

(b) the coordinates of the y-intercept of the graph of $y = \log_{10}(x + 100)$. (2 marks)

$y = \log_{10}(10^2)$
$= 2\log_{10} 10 = 2$

At (0, 2)

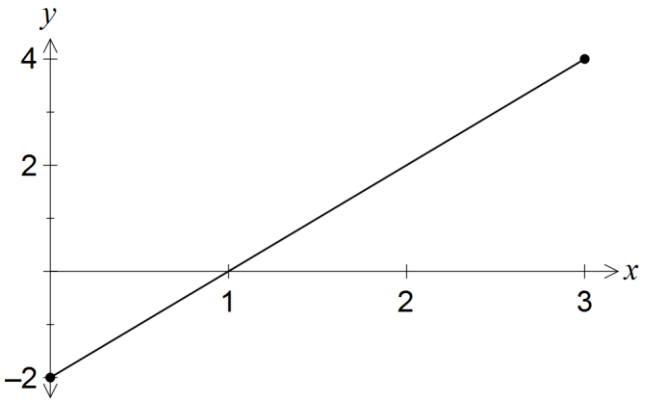
(c) the coordinates of the x-intercept of the graph of $y = 2 - \log_e x$. (2 marks)

$0 = 2 - \ln(x) \Rightarrow 2 = \ln(x) \Rightarrow x = e^2$

At (e^2, 0)

Question 5 **(8 marks)**

The graph of $y = f(x)$, where $f(x) = 2x - 2$, is shown below for $0 \le x \le 3$.



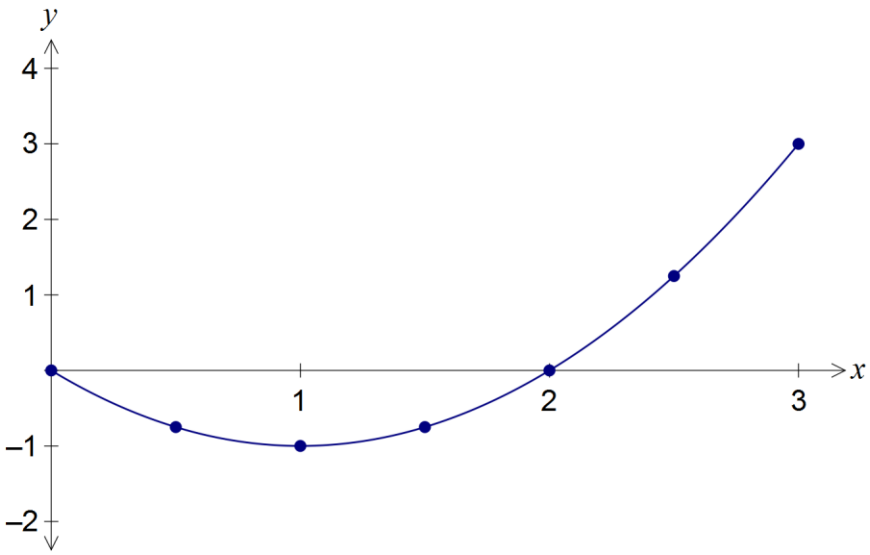
The function $A(x)$ is defined by $A(x) = \int_0^x f(t)\,dt$, for $0 \le x \le 3$.

(a) State the values of x for which $A(x)$ is increasing. (1 mark)

$x > 1$

(b) Complete the table below and use it to sketch the graph of $y = A(x)$ for $0 \le x \le 3$ on the axes provided. (4 marks)

x	0	0.5	1	1.5	2	2.5	3
$A(x)$	0	-0.75	**-1**	-0.75	**0**	1.25	3



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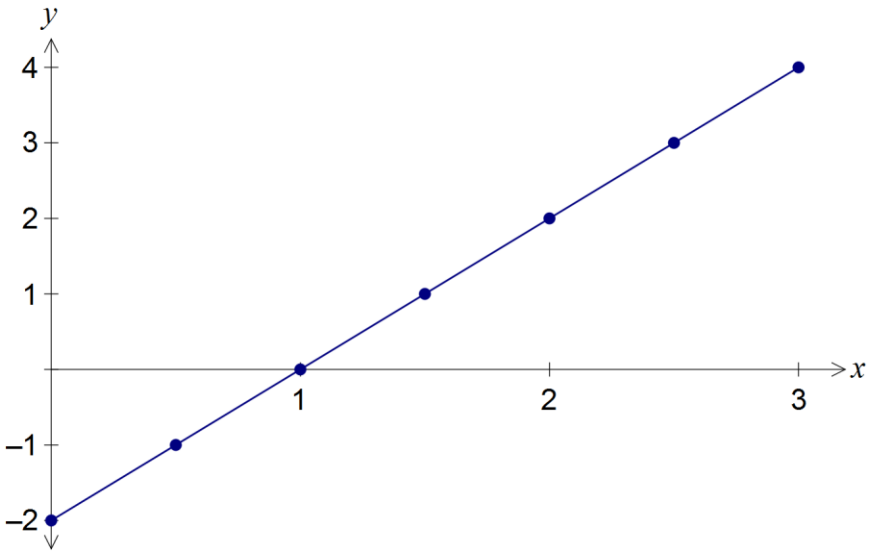
(c) Use your graph from (b) to sketch the graph of $y = A'(x)$ on the axes below. (2 marks)



(d) State the defining rule for $A'(x)$. (1 mark)

$A'(x) = 2x - 2$

Question 6 **(5 marks)**

(a) Write, where possible, the following as single integrals. If not possible, explain why.

(i) $\int_2^3 \left(e^x + 2\right)\, dx + \int_3^5 \left(e^x + 2\right)\, dx$. (1 mark)

$$\int_2^5 \left(e^x + 2\right)\, dx$$

(ii) $\int_0^{\pi} \left(\cos\theta - \sin\theta\right)\, d\theta + 2\int_0^{\pi} \left(\cos\theta - \sin\theta\right)\, d\theta$. (1 mark)

$$3\int_0^{\pi} \left(\cos\theta - \sin\theta\right)\, d\theta$$

(b) Evaluate $\int_5^8 \left(x^3 - x^2 + 3x - 1\right)\, dx + \int_8^5 \left(x - x^2 + x^3 - 1\right)\, dx$. (3 marks)

$$= \int_5^8 \left(x^3 - x^2 + 3x - 1\right)\, dx - \int_5^8 \left(x - x^2 + x^3 - 1\right)\, dx$$

$$= \int_5^8 \left(2x\right)\, dx$$

$$= \left[x^2\right]_5^8 = 64 - 25 = 39$$

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Question 7 **(5 marks)**

(a) Simplify $\dfrac{3\log 100}{4\log 1000}$. (2 marks)

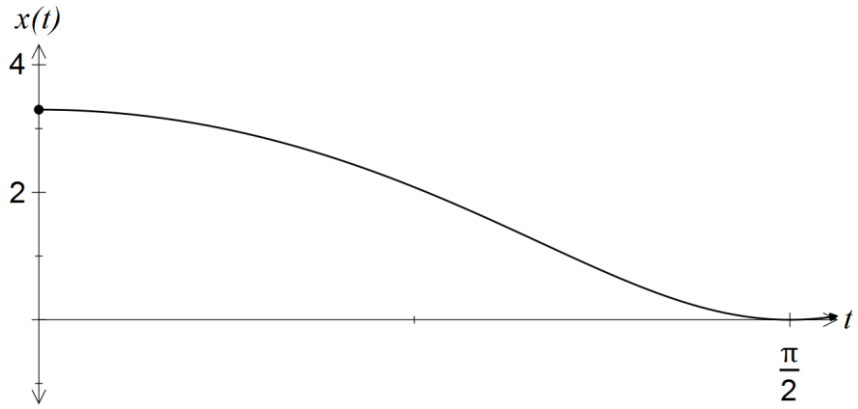
$$\frac{3\times 2\log 10}{4\times 3\log 10}=\frac{1}{2}$$

(b) If $\log_a(x)=0.313$, determine $\log_a\left(\dfrac{1}{x^2}\right)$. (3 marks)

$$\log_a \frac{1}{x^2}=\log_a x^{-2}$$
$$=-2\log_a x$$
$$=-2(0.313)$$
$$=-0.626$$

Question 8 **(10 marks)**

The position of a particle relative to a fixed point O, t seconds after motion began, is given by the function $x(t) = 3\ln\left(\cos(2t) + 2\right)$, where x is measured in metres, and is shown in the graph below.



(a) Determine the velocity function for the particle. (3 marks)

$$v(t) = x'(t)$$

$$= \frac{3(-2\sin(2t))}{\cos(2t) + 2}$$

$$= \frac{-6\sin(2t)}{\cos(2t) + 2}$$

(b) Determine the initial acceleration of the particle. (4 marks)

$$a(t) = x''(t)$$

$$= \frac{\left(-12\cos(2t)\right)\left(\cos(2t) + 2\right) - \left(-6\sin(2t)\right)\left(-2\sin(2t)\right)}{\left(\cos(2t) + 2\right)^2}$$

$$a(0) = \frac{-12 \times 3 - 0 \times 0}{3^2} = -4 \text{ m/s}^2$$

End of questions

(c) Evaluate $\int_0^{\pi} x'(t)\ dt$ and interpret your answer in the context of this question. (3 marks)

$$\begin{aligned}\int_0^{\pi} x'(t)\ dt &= \left[x(t)\right]_0^{\pi} \\ &= 3\ln(\cos(2\pi)+2) - 3\ln(\cos(0)+2) \\ &= 3\ln 3 - 3\ln 3 \\ &= 0\end{aligned}$$

Change in displacement of particle during first π seconds of motion is zero, so must be at initial position.

Additional working space

Question number: _____

Additional working space

Question number: _____